Aegis: A Dynamic Risk Control Framework Based on the Volatility Pressure Index (VPI)

A Simple Law of Variance Dynamics

Toshiharu Honda
Quant Finance Lab Kagoshima

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Abstract

This study introduces the Volatility Pressure Index (VPI), defined as the logarithmic change in variance, and proposes a simple framework for describing volatility dynamics. While conventional volatility research has primarily focused on levels and forecasting, this study treats the rate of change of variance itself as a state variable.

Theoretically, we show that variance is determined by the cumulative sum of VPI, and that abrupt volatility fluctuations can be understood as the interaction between the accumulated state and its changes. Based on this structure, we propose a dynamic risk control system, **Aegis**, which utilizes VPI-derived indicators. Aegis sequentially evaluates market conditions and continuously adjusts exposure.

A forward test on the U.S. equity market (SPY) shows that Aegis mitigates drawdowns during major market downturns while maintaining participation during recovery phases.

This study provides a framework that views volatility not as a static statistical quantity, but as a dynamic process described by state and change.

Keywords: Volatility Pressure Index (VPI), Variance Dynamics, Risk Management, Aegis System, Econophysics, Dynamic Asset Allocation

JEL Classification: G11, G12, G17, C02

1 Introduction

Volatility in financial markets continues to play a central role in both risk management and asset pricing. Traditional research has primarily focused on the level of volatility and its predictability, leading to the development of various modeling approaches. Representative examples include the GARCH model, which captures time-varying conditional variance [3], and realized volatility, which improves measurement accuracy by utilizing high-frequency data as an observable quantity [2]. In addition, the variance risk premium, which evaluates risk based on the discrepancy between expected and realized variance in option markets, has been extensively studied [4].

In recent years, there has been a growing interest in reinterpreting financial markets as physical dynamic systems, aiming to explain structural variations in momentum and volatility. For instance, recent studies have explored scale-invariant dynamical systems underlying market momentum [5] and approaches that apply concepts from Newtonian mechanics to capture market instability [8]. While these studies are important for describing market instability and regime changes, they often rely on complex modeling frameworks or are inherently dependent on specific volatility levels.

At the same time, alternative perspectives exist that interpret volatility dynamics in terms of information-geometric distances or rough properties [1, 6]. However, a simple and direct framework that treats the rate of change of variance itself as a fundamental state variable, and describes volatility dynamics through its cumulative structure, has not yet been fully established.

This study addresses this gap by introducing the Volatility Pressure Index (VPI), defined as the logarithmic change in variance. VPI is a simple measure of the growth rate of variance, and its cumulative structure allows for a direct description of the temporal evolution of volatility. The central premise of this study is that volatility should not be viewed merely as a level to be predicted, but rather as a state variable that accumulates and releases over time, analogous to physical "pressure" [10]. In particular, we show that the cumulative rate of change in variance characterizes the state of the market, and that its variation is closely associated with sudden volatility spikes.

Furthermore, based on this framework, we propose a risk control system, **Aegis**, which utilizes VPI-derived state variables. While conventional volatility-targeting strategies [7, 9] have been effective in improving risk-adjusted returns, Aegis aims to more sensitively capture changes in the market's dynamic "pressure," thereby achieving both drawdown reduction and sustained market participation.

The remainder of this paper is organized as follows. Section 2 introduces the definition of VPI. Sections 3 and 4 present its theoretical properties. Section 5 examines its empirical characteristics. Section 6 introduces the Aegis risk control system. Section 7 reports the empirical results, and Section 8 provides discussion. Finally, Section 9 concludes.

2 Definition of the Volatility Pressure Index

In this section, we define the Volatility Pressure Index (VPI), which serves as the fundamental measure in this study.

In financial markets, volatility is typically defined as the square root of the variance of returns. Let Var_t $(= \sigma_t^2)$ denote the variance at time t. Focusing on its temporal variation, we introduce the following state variable:

$$VPI_t = \log Var_t - \log Var_{t-1} \tag{1}$$

By this definition, VPI represents the logarithmic change in variance, that is, the growth rate of variance.

2.1 Intuitive Interpretation

As is evident from Equation (1), VPI captures both the direction and the magnitude of changes in variance. Specifically, the sign of VPI indicates the direction of change, while its absolute value reflects the strength of that change.

Furthermore, by taking logarithmic differences, VPI naturally expresses relative changes in variance. This property allows for meaningful comparisons across different volatility regimes.

2.2 Role of the Logarithmic Transformation

The use of the logarithmic transformation for variance is motivated by the following considerations.

First, since variance is strictly positive, taking logarithms enables its changes to be treated additively. This facilitates a clear representation of the temporal accumulation structure.

Second, logarithmic changes are scale-invariant. As a result, the analysis focuses not on the absolute level of volatility, but on its rate of change.

2.3 Basic Properties

Equation (1) can be rewritten in the following form:

$$Var_t = Var_{t-1} \cdot \exp(VPI_t) \tag{2}$$

Equation (2) shows that the time evolution of variance is governed by VPI. Thus, VPI is not merely an auxiliary indicator, but a state variable that directly characterizes the dynamics of variance.

2.4 Perspective of This Study

Conventional volatility research has focused primarily on the level of variance or its prediction. In contrast, this study treats the rate of change of variance itself as the fundamental state variable.

From this perspective, volatility is not a static level, but a dynamic process that evolves over time. VPI provides a simple yet powerful state variable that captures the dynamic structure of volatility, and all theoretical developments in this study are built upon it.

2.5 Summary

In this section, we defined VPI as the logarithmic change in variance and presented its fundamental properties. VPI is a simple state variable representing the growth rate of variance, and its temporal accumulation enables a direct description of volatility dynamics.

In the next section, we derive theoretical relationships governing the time evolution of variance based on this cumulative structure.

3 Theoretical Framework

In this section, we derive the theoretical relationships governing the time evolution of variance based on the Volatility Pressure Index (VPI) defined in the previous section.

3.1 Cumulative Structure

By recursively applying Equation (2), variance can be expressed as:

$$Var_t = Var_0 \cdot \exp\left(\sum_{i=1}^{t} VPI_i\right) \tag{3}$$

Equation (3) shows that the level of variance is determined by the cumulative sum of VPI.

That is, variance should be understood not as a result of a single-period change, but as the accumulation of past growth rates.

3.2 Extension to Continuous Time

The discrete-time formulation can be naturally extended to continuous time. Let $Var(t)$ denote variance as a continuous function of time. Then, VPI is defined as:

$$VPI(t) = \frac{d}{dt} \log Var(t) \tag{4}$$

This expression indicates that VPI represents the instantaneous growth rate of variance.

3.3 Differential Equation Representation

Equation (4) can be rewritten as:

$$\frac{dVar(t)}{dt} = VPI(t) \cdot Var(t) \tag{5}$$

Equation (5) is the fundamental equation describing the time evolution of variance driven by VPI.

3.4 Integral Form

Solving Equation (5), variance can be expressed as:

$$Var(t) = Var(0) \cdot \exp\left(\int_0^t VPI(s)\, ds\right) \tag{6}$$

This solution corresponds to the continuous-time analogue of the cumulative structure in discrete time (Equation (3)).

3.5 Theoretical Implications

The above results demonstrate that VPI is a fundamental state variable that directly governs the dynamics of variance, and hence volatility.

In particular, VPI represents the growth rate of variance, and its cumulative effect determines the level of variance, that is, the level of volatility.

Under this framework, volatility is not a static quantity, but a dynamic process that evolves over time.

3.6 Summary

In this section, we derived the time evolution of variance based on VPI in both discrete-time and continuous-time settings.

The results show that variance is determined by the cumulative effect of VPI, and this cumulative structure constitutes the theoretical foundation of this study.

In the next section, we further examine this structure and analyze the time-dependent properties of variance.

4 Interpretation of the Cumulative Structure

In this section, we further deepen the interpretation of the cumulative structure of variance derived in Section 3. As shown in Equation (6), variance is determined by the cumulative effect of VPI. That is, the level of volatility depends not only on information at a single point in time, but also on its historical path.

Moreover, Equation (6) implies that variance follows an exponential growth process, with VPI representing its growth rate. When VPI is positive, variance increases; when it is negative, variance decays.

4.1 Accumulation of State

Consider the cumulative term in Equation (6):

$$A(t) = \int_0^t VPI(s)\, ds \tag{7}$$

The quantity $A(t)$ represents the accumulated history of variance growth. When $A(t)$ is large, variance has already grown to a high level, indicating that the market is in a high-volatility state. Conversely, when $A(t)$ is small, the market can be interpreted as relatively stable.

A key implication of this cumulative structure is that instability accumulates over time. If VPI remains positive over a sustained period, variance grows persistently, and the market transitions into a more unstable state. In such a state, even small shocks can lead to large price fluctuations.

4.2 Volatility as a Dynamic State Variable

From the above considerations, the VPI-based volatility model depends on both the cumulative state $A(t)$ and the current value of VPI. At the same time, changes in VPI alter this accumulated state.

In particular, an increase in VPI corresponds to an increase in the growth rate of variance, while a decrease corresponds to a reduction in that growth rate. Therefore, abrupt changes in volatility can be understood as the result of interactions between the accumulated state and its incremental changes.

While conventional volatility research has focused primarily on the level of volatility, this study jointly considers both the cumulative effect of VPI and its changes. This perspective enables a deeper understanding of volatility dynamics.

Importantly, this viewpoint naturally extends beyond descriptive analysis to a control problem in risk management. In the next section, we examine the empirical properties of VPI based on this framework.

5 Empirical Properties of VPI

In this section, we summarize the empirical properties of the Volatility Pressure Index (VPI) and clarify its relationship with the volatility dynamics discussed in the previous section.

5.1 Distributional Characteristics of VPI

Since VPI is defined as the logarithmic change in variance, its distribution is centered around zero. Empirical market data also show that VPI tends to be concentrated near zero (Figure 1).

This property is consistent with the tendency of volatility to remain within a bounded range over the long run. It also indicates that VPI captures not the level of volatility, but the direction and magnitude of its changes.

5.2 Variance Dynamics and the Cumulative Structure

As shown in Section 4, variance can be expressed as the time integral of VPI, that is, as the cumulative sum of VPI. Therefore, understanding abrupt changes in volatility requires focusing not only on instantaneous VPI, but also on the level formed by its accumulation.

If VPI remains positive over a certain period, the growth rate of variance is persistently positive, and the cumulative value increases. In this case, the market

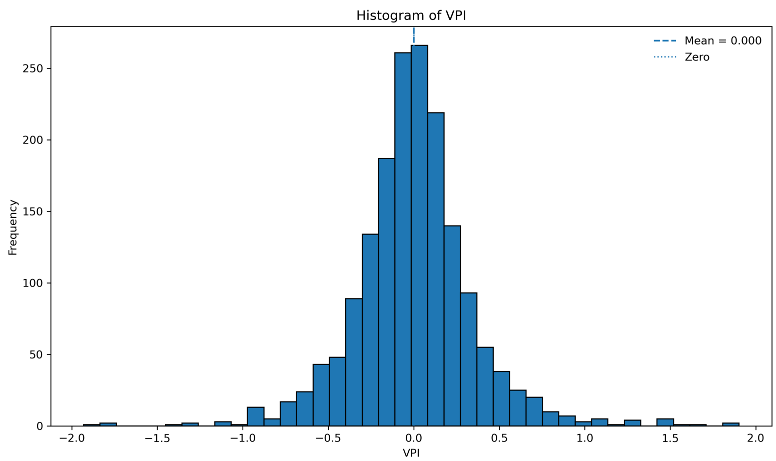

Figure 1: Histogram of VPI based on weekly data for SPY. The distribution is centered around zero, indicating that increases and decreases in log-variance are approximately balanced over the long run.

transitions into a state where volatility expansion becomes more likely. Even small fluctuations in VPI may then disrupt equilibrium and trigger large volatility movements.

Figure 2 shows the relationship between binned cumulative VPI and future variance. While future variance remains relatively stable in low and medium ranges of cumulative VPI, it increases sharply in high ranges.

This result suggests that sudden expansions in variance depend on the level of VPI as a cumulative state variable.

An additional key factor is the temporal change in VPI itself. When VPI increases rapidly, the growth rate of variance rises, leading to an abrupt expansion of volatility.

Therefore, large volatility movements can be explained by a two-layer structure: (i) a high accumulated growth state driven by cumulative VPI, and (ii) rapid changes in VPI.

5.3 Relationship with Tail Risk

When the cumulative VPI is large, abrupt expansions in volatility become more likely, leading to increased tail risk.

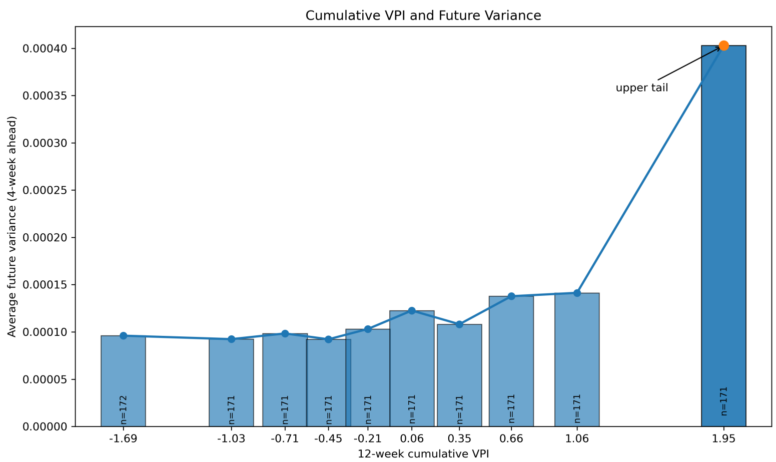

Figure 2: Binned relationship between 12-week cumulative VPI and 4-week-ahead variance for SPY. Future variance remains stable in low and medium ranges of cumulative VPI, but increases sharply in high ranges.

Thus, VPI can be interpreted not merely as the rate of change of variance, but as a state variable that reflects the likelihood of extreme market movements.

This interpretation implies that evaluating market risk requires considering both the current state (cumulative VPI) and its changes (variations in VPI), rather than relying solely on the level of volatility.

Consequently, a control framework that dynamically adjusts market exposure based on these factors naturally emerges.

6 Risk Control System (Aegis)

In this section, we introduce a risk control system, **Aegis**, which dynamically adjusts market exposure based on the VPI dynamics discussed in the previous sections.

VPI represents the growth rate of variance, while its cumulative value characterizes the state of the market. At the same time, its temporal changes are closely associated with abrupt fluctuations in volatility. Therefore, effective risk management requires considering both the accumulated growth state and changes in the growth rate.

Aegis integrates these components into a unified framework that dynamically adjusts market exposure in response to evolving market conditions.

6.1 Energy Measure (Cumulative State)

We first define a state variable E_t based on the cumulative VPI:

$$E_t = \sum_{i=t-L}^{t} VPI_i$$

where L denotes the lookback window. The variable E_t represents the accumulated growth rate of variance at time t, capturing the extent to which the market is in a volatility expansion state.

6.2 State Dynamics

Next, we define the temporal change in the energy measure:

$$\Delta E_t = E_t - E_{t-1}$$

The quantity ΔE_t captures changes in the growth state, indicating whether cumulative VPI is increasing (destabilizing) or decreasing (stabilizing).

6.3 Score Function and Exposure Mapping

To integrate the cumulative state and its change, we introduce the following score function:

$$S_t = z(E_t) + \lambda \cdot z(\Delta E_t)$$

where $z(\cdot)$ denotes a standardization function and λ is a weighting parameter. This formulation captures both the level and the dynamics of the market state.

To map the score S_t to market exposure w_t, we use a monotonic nonlinear function (sigmoid):

$$w_t = w_{\min} + (1 - w_{\min}) \cdot \frac{1}{1 + e^{-k(S_t - \theta)}}$$

where $w_{\min}$ is the minimum exposure level, k is a sensitivity parameter, and θ is a threshold. Under this transformation, market exposure is reduced continuously as instability increases.

6.4 Asymmetric Control and Re-entry Conditions

Aegis incorporates asymmetry in exposure adjustment. Specifically, exposure reduction is executed rapidly, while recovery is performed more gradually.

This design suppresses excessive re-entry caused by short-term noise and improves risk control under unstable market conditions.

Re-entry into the market is governed by three conditions: (i) a decline in the cumulative state (E_t decreases), (ii) stabilization in the state dynamics (ΔE_t decreases), and (iii) recovery in price trends.

This ensures that exposure is restored only after structural stabilization, rather than temporary reductions in volatility.

6.5 Sequential Implementation

Aegis operates sequentially. Decisions at each time step are made using only information available up to that point.

This property ensures that the system does not rely on future information and is therefore implementable in real-world settings.

6.6 Summary

Aegis is a risk control system that captures dynamic market risk by integrating the cumulative state of VPI and its changes, and continuously adjusts exposure accordingly.

This framework enhances resilience to volatility spikes while maintaining participation in the market.

7 Empirical Analysis

In this section, we evaluate the effectiveness of the Aegis risk control system using real market data. We focus on SPY, a representative index of the U.S. equity market, and assess performance based on a forward-testing framework.

The forward test is conducted using a sequential decision-making process designed to replicate real-world implementation, ensuring that no future information is used.

7.1 Data and Setup

The analysis is based on daily closing prices of SPY. Returns are computed as log returns, and variance is estimated using historical data.

The key components of Aegis include VPI, the cumulative state variable E_t, the state change ΔE_t, the score function S_t, and the exposure w_t.

To avoid overfitting, model parameters (e.g., lookback window, standardization method, and sensitivity coefficients) are selected based on generally reasonable values rather than optimization.

7.2 Forward Test Design

The forward test follows a sequential procedure at each time step:

1. Compute VPI using past data

2. Update the cumulative state E_t and its change ΔE_t

3. Compute the score S_t

4. Determine the exposure w_t

5. Apply the decision to the next period

This process is executed iteratively, with all decisions based solely on information available at each point in time.

7.3 Performance Comparison

We compare the performance of Aegis combined with trend following (Aegis + TF) against two benchmarks: Buy-and-Hold (SPY) and a standalone trend-following strategy (TF).

The results exhibit the following characteristics:

- Losses are mitigated during market downturns
- Volatility is reduced
- Risk-adjusted returns are improved

In particular, during sharp market declines, exposure is automatically reduced, leading to significant drawdown mitigation (Table 1, Figure 3).

Table 1: Performance comparison based on weekly data from 2000 to 2026. Aegis reduces maximum drawdown and volatility while improving risk-adjusted returns compared to both standalone trend following and SPY (Buy-and-Hold).

Metric	**Aegis + TF**	**TF**	**SPY (Buy & Hold)**
Annual Geometric Return	13.20%	8.40%	9.32%
Annualized Volatility	11.22%	13.92%	17.79%
Max Drawdown	-22.24%	-31.83%	-54.61%
Downside Deviation	6.78%	9.71%	12.47%
Sharpe Ratio	1.16	0.65	0.59
Sortino Ratio	1.92	0.93	0.84
Calmar Ratio	0.59	0.26	0.17

7.4 Drawdown Characteristics

A key feature of Aegis is not merely return enhancement, but effective loss control. Focusing on maximum drawdown and downside deviation, Aegis significantly reduces risk during periods of market stress.

As shown in Figure 4, Aegis effectively limits drawdowns during sharp market declines, thereby avoiding prolonged recovery periods.

7.5 Dynamic Exposure Behavior

Exposure under Aegis adjusts continuously in response to market conditions. During stable periods, exposure remains high, while it declines rapidly as instability increases.

This dynamic adjustment allows the system to maintain market participation while avoiding excessive risk-taking.

As illustrated in Figure 5, Aegis does not maintain constant exposure. Instead, it selectively reduces exposure during periods of instability.

At the same time, exposure quickly returns to full levels during recovery phases, allowing the strategy to limit exposure to negative returns while efficiently capturing positive returns.

7.6 Interpretation

These results indicate that the VPI-based state variables effectively capture and respond to changes in market risk conditions.

In particular, the cumulative state reflects the buildup of instability, while changes in the state provide early signals of regime transitions.

The empirical results demonstrate that Aegis adapts to dynamic market conditions and simultaneously achieves drawdown reduction and improved risk-adjusted returns.

These findings support the view that the VPI-based Aegis framework is not only theoretically consistent, but also practically effective.

8 Discussion

In this section, we discuss the theoretical and practical implications of VPI and Aegis based on the empirical results presented in the previous section.

8.1 Interpretation of VPI as a State Variable

Although VPI is formally defined as the logarithmic change in variance, its role extends beyond a simple growth rate.

As demonstrated in the empirical analysis, VPI possesses two complementary aspects: (i) a local indicator representing the instantaneous growth rate of variance, and (ii) a cumulative state variable that characterizes the market condition.

In particular, the cumulative state reflects the accumulation of market instability and indicates conditions under which abrupt volatility expansions are more likely to occur.

In this sense, VPI differs fundamentally from conventional volatility measures, as it provides a description of the underlying *state* of the market.

8.2 Comparison with Conventional Approaches

Traditional volatility models focus on time-series frameworks such as GARCH or on implied volatility derived from option pricing models such as Black–Scholes.

In contrast, the approach proposed in this study treats the rate of change of variance itself as the fundamental state variable.

This distinction enables VPI to offer a model-free, structurally simple representation with strong interpretability, as well as direct applicability to control problems.

8.3 Why the System Works: Mechanism

The effectiveness of Aegis originates from the structural properties of VPI.

Since variance is determined by the cumulative effect of VPI, a sustained positive VPI implies persistent growth in volatility, leading the market toward an unstable regime.

In such a state, even small changes in VPI can trigger large price fluctuations.

Moreover, when VPI itself changes rapidly, the growth rate of variance also shifts, resulting in abrupt volatility expansions.

Aegis leverages this structure by capturing the accumulation of risk through the cumulative state and detecting regime transitions through changes in VPI, thereby adjusting exposure at appropriate times.

8.4 Importance of Asymmetry

The empirical results suggest that asymmetric control of exposure is crucial.

Specifically, responding rapidly to increases in risk while reacting more cautiously to decreases in risk contributes significantly to drawdown reduction.

This asymmetry is consistent with the empirical observation that market instability tends to rise quickly but dissipates more gradually.

8.5 Practical Implications

The findings of this study provide the following practical insights:

· Risk cannot be fully captured by volatility levels alone

· Both the cumulative state and its changes must be monitored simultaneously

· Market exposure should be adjusted continuously rather than discretely

Aegis provides a simple and implementable framework that satisfies these requirements.

9 Conclusion

This study introduced the Volatility Pressure Index (VPI), defined as the logarithmic change in variance, and proposed a simple framework for describing volatility dynamics.

VPI represents the growth rate of variance, and its cumulative structure determines the level of volatility. This relationship implies that abrupt changes in volatility are driven not by single-period fluctuations, but by the interaction between accumulated states and their changes.

Building on this structure, we developed a risk control system, **Aegis**, based on VPI-derived state variables. Aegis integrates the cumulative state and its changes to capture dynamic market risk and continuously adjust exposure.

Empirical analysis shows that Aegis reduces drawdowns during market downturns while maintaining long-term participation in the market. These results indicate that the VPI-based framework possesses both theoretical consistency and practical effectiveness.

The main contributions of this study can be summarized as follows:

1. A simple dynamic model based on the rate of change of variance

2. An interpretation of VPI as a state variable describing market conditions

3. An implementable risk control framework based on this structure

The analysis in this paper focuses on SPY, and further validation across other asset classes remains an important direction for future research.

While the current parameter settings are chosen to avoid overfitting, more systematic approaches to parameter selection may further improve performance.

In addition, although this study focuses on risk control, integrating the framework with asset allocation strategies such as momentum and carry represents a promising avenue for future work.

Finally, this study provides a perspective that views volatility not as a static statistical quantity, but as a dynamic state evolving over time. This perspective opens new possibilities for both understanding financial markets and designing effective risk management systems.

9.1 Future Research

The VPI introduced in this study describes the cumulative growth dynamics of a single asset. However, it can be naturally extended to a multidimensional framework representing information flow across multiple assets.

In particular, extending the framework to a vector field representation of information dynamics—where the difference between realized and expected variance acts as a potential—suggests a fluid-dynamics-like formulation.

Such an approach may provide a new way to quantify the speed of information transmission across markets and the notion of "informational distance," potentially leading to a Navier–Stokes-type formulation for financial systems.

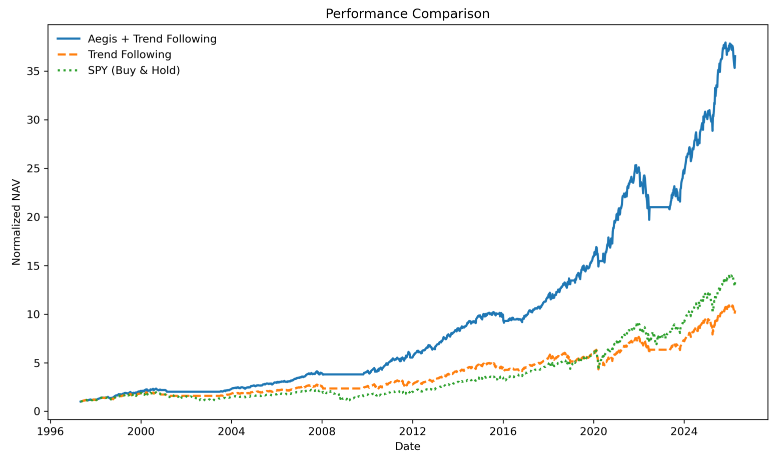

Figure 3: Cumulative returns based on weekly data from 2000 to 2026. Compared with standalone trend following and SPY (Buy-and-Hold), Aegis achieves substantial drawdown reduction while improving risk-adjusted returns.

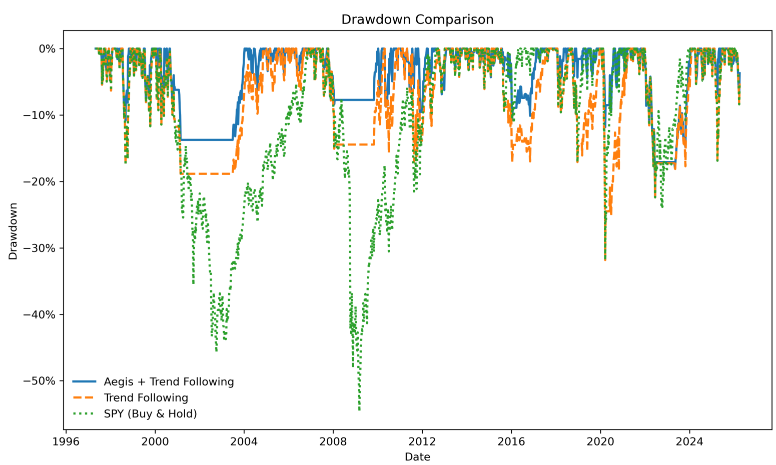

Figure 4: Drawdown trajectories based on weekly data from 2000 to 2026. Aegis consistently limits the depth of drawdowns compared with standalone trend following and SPY (Buy-and-Hold), with particularly strong effects during periods of market stress.

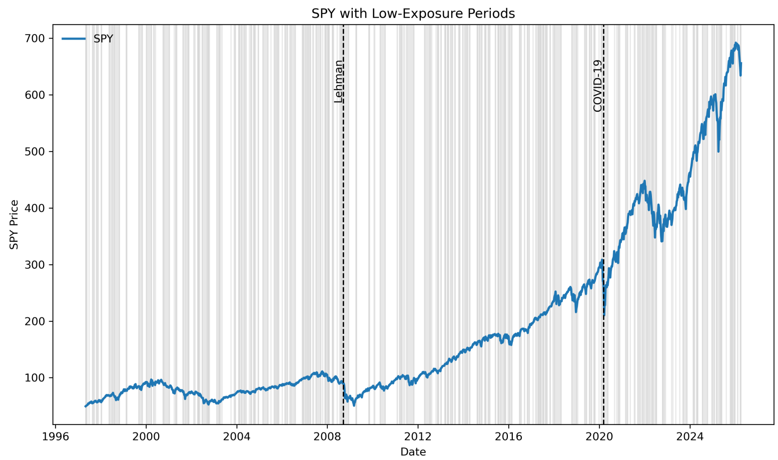

Figure 5: SPY price dynamics with low-exposure periods identified by Aegis, based on weekly data from 2000 to 2026. Shaded regions indicate periods of reduced exposure, showing defensive adjustments around major stress events such as the Lehman shock and the COVID-19 shock.

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